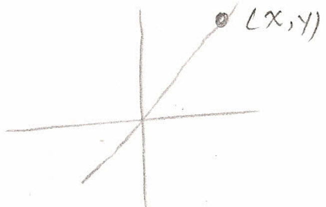
MTH 295
Fall 2019
Homework 10
Due Tuesday 11/26

Name: Key

1) a) Find the equations of all straight line trajectories of the planar system
$x' = y, y' = 3x - y$.

There is a fixed point at (0,0). Suppose (x,y) lies on a straight line solution, then the slope m is given by



$$m = \frac{y}{x} = \frac{dy}{dx} = \frac{y'}{x'} = \frac{3x-y}{y} = \frac{3 - y/x}{y/x} = \frac{3-m}{m}$$

so $m^2 = 3 - m$

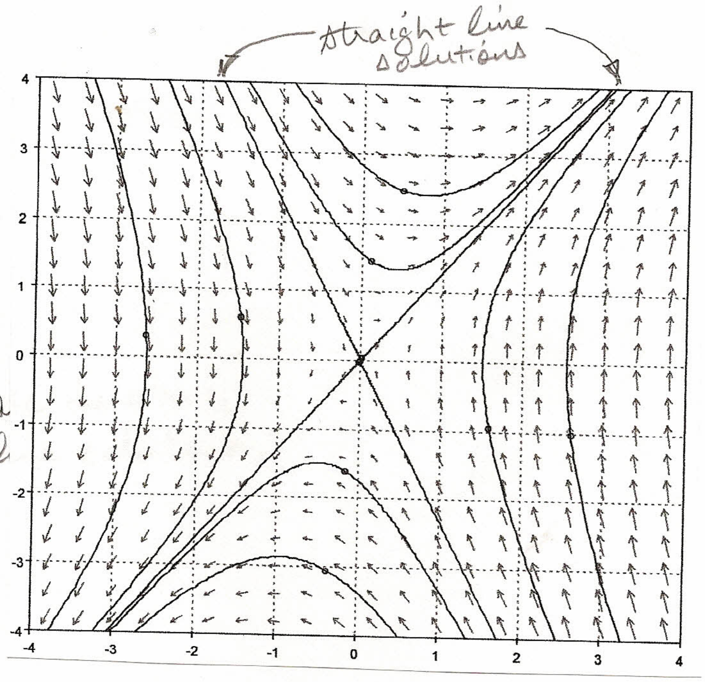
$m^2 + m - 3 = 0$

$m = \frac{-1 \pm \sqrt{1+12}}{2}$

$= \frac{-1 \pm \sqrt{13}}{2}$

so $y = \frac{-1+\sqrt{13}}{2} x$

and $y = \frac{-1-\sqrt{13}}{2} x$



b) Attach the phase diagram of the system. Label the trajectories you found in part a) and include a few more trajectories to illustrate the behavior.

c) Is the origin a sink, source, or saddle point.

saddle

d) What can you say about the eigenvalues of this system without solving it?

both real, one positive and one negative.

2) For each of the following, find all equilibrium solutions and determine if they are sinks, sources, or neither. Sketch the phase line and be sure to include the flows. Note that $x = x(t)$. Attach the direction field to show that it agrees with your phase line.

a) $x' = x^3 - 3x$

$x' = x(x^2-3) = 0$

$x = 0, \sqrt{3}, -\sqrt{3}$ are fixed points

if $x > \sqrt{3}$ then $x' > 0$

if $0 < x < \sqrt{3}$ then $x' < 0$

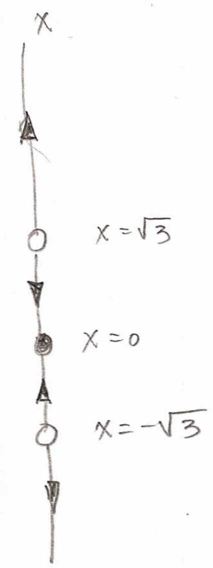
if $-\sqrt{3} < x < 0$ then $x' > 0$

if $x < -\sqrt{3}$ then $x' < 0$

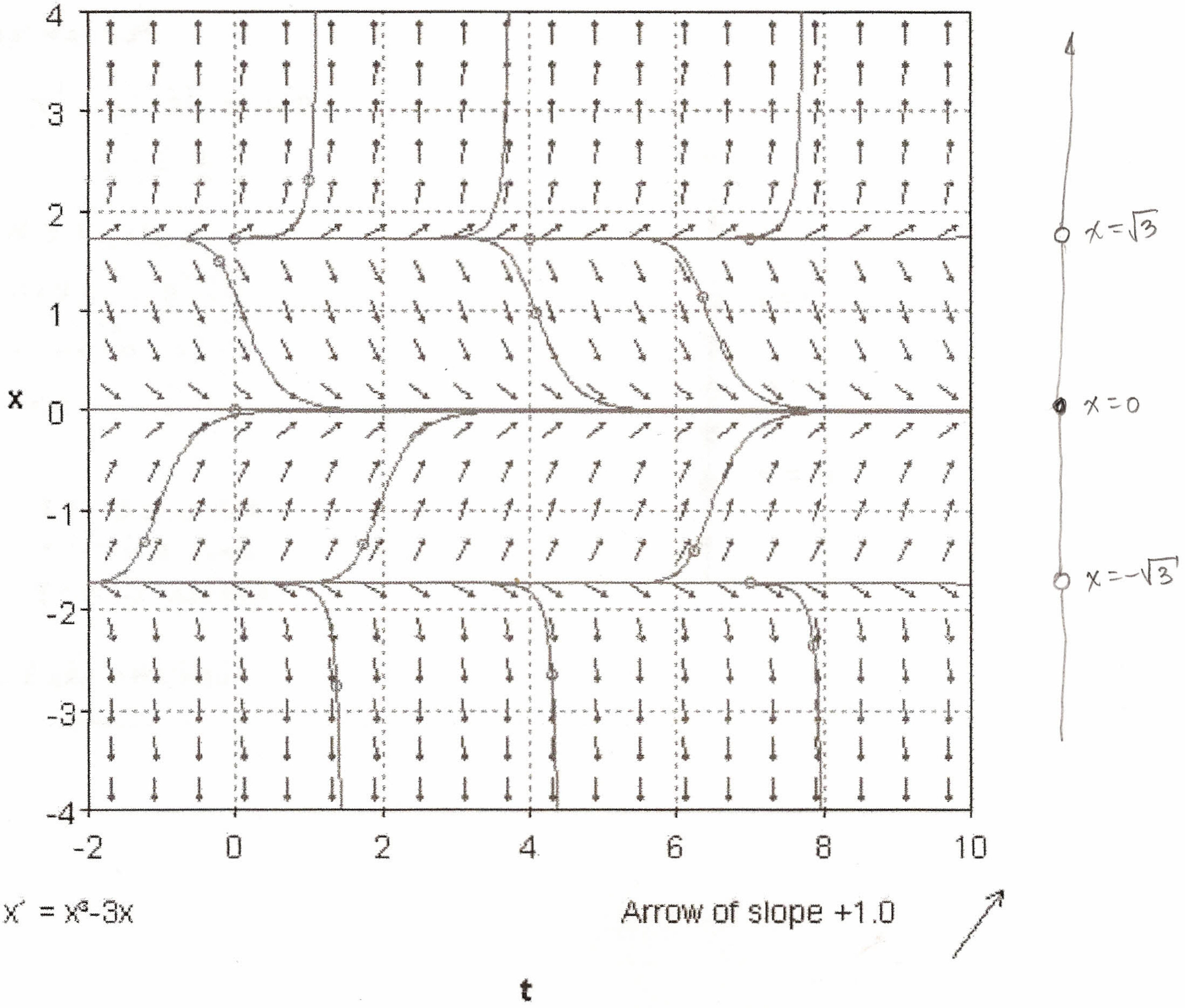
so $x = \sqrt{3}$ is a source

$x = 0$ is a sink

$x = -\sqrt{3}$ is a source



D-field attached



2A

b) $x' = x^4 - x^2$

$$x' = x^2(x^2-1) = 0$$

$$x = 0, \pm 1$$

if $x > 1$ $x' > 0$

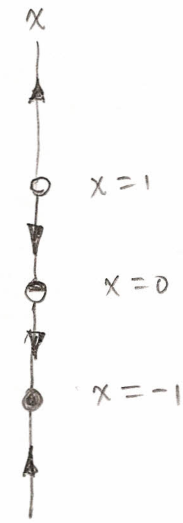
if $0 < x < 1$, $x' < 0$

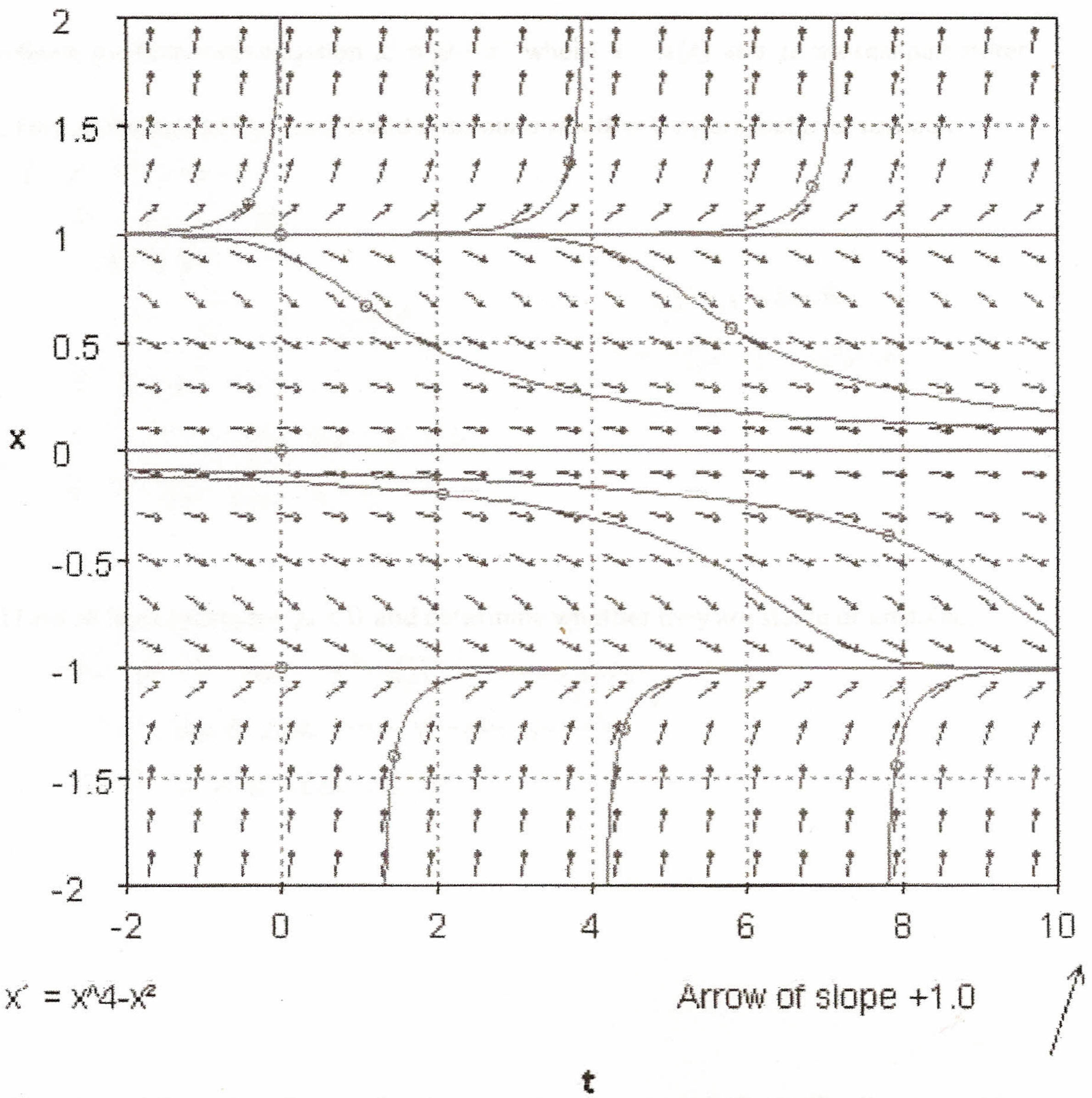
if $-1 < x < 0$, $x' < 0$

if $x < -1$, $x' > 0$

so $x = 1$ is a source
$x = -1$ is a sink
$x = 0$ is neither

D-field attached





2B

3) Given the first order equation $x' = \mu - x^2$ where $x = x(t)$ and μ is a real parameter

a) Find all fixed points for $\mu \geq 0$ and determine whether they are stable or unstable.

if $\mu > 0$ then

$x' = \mu - x^2 = 0$

$x^2 = \mu$

$x = \pm\sqrt{\mu}$ are fixed

if $x > \sqrt{\mu}$ then $x' < 0$

if $-\sqrt{\mu} < x < \sqrt{\mu}$ then $x' > 0$

if $x < -\sqrt{\mu}$ then $x' < 0$

so $x = \sqrt{\mu}$ is stable

$x = -\sqrt{\mu}$ is unstable

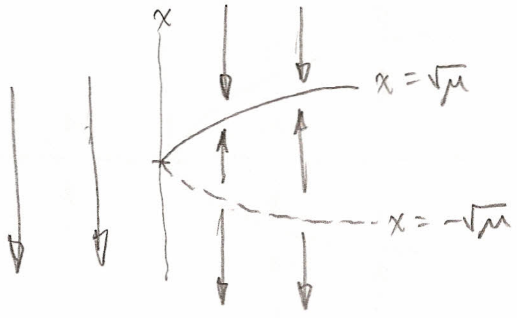
b) Find all fixed points for $\mu < 0$ and determine whether they are stable or unstable.

if $\mu < 0$ then $x^2 = \sqrt{\mu}$ is imaginary

and there are no fixed points

and $x' < 0$ for all x.

c) Sketch the bifurcation diagram for this equation. Be sure to indicate the direction of flow on your diagram (with arrows). This is called a "blue sky bifurcation" because the fixed points appear out of nowhere.



4) Given the first order equation $x' = \mu x - x^3$ where $x = x(t)$ and μ is a real parameter.

a) Find all fixed points for $\mu \geq 0$ and determine whether they are stable or unstable.

$\mu x - x^3 = x(\mu - x^2) = 0$

so if $\mu > 0$ there are fixed points at $x = 0$, $x = \pm\sqrt{\mu}$

if $x > \sqrt{\mu}$, $x' < 0$

$0 < x < \sqrt{\mu}$, $x' > 0$

$-\sqrt{\mu} < x < 0$, $x' < 0$

$x < -\sqrt{\mu}$; $x' > 0$

so $x = \sqrt{\mu}$ is stable

$x = 0$ is unstable

$x = -\sqrt{\mu}$ is stable

b) Find all fixed points for $\mu < 0$ and determine whether they are stable or unstable.

if $\mu < 0$ then $x = \pm\sqrt{\mu}$ is imaginary and there is one fixed point at $x = 0$.

if $x > 0$ then $x' < 0$

if $x < 0$ then $x' > 0$

so $x = 0$ is stable.

c) Sketch the bifurcation diagram for this equation. Be sure to indicate the direction of flow on your diagram (with arrows). This is called a "pitchfork bifurcation" for obvious reasons.